Evaluation of the LOCO program on the SPS

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- **What is LOCO and how does it work ?**
- **Test of LOCO on SPS data**
- **Summary and MD requests**

LOCO principle (i)

- The **LOCO** program (**Linear Optics from Closed Orbits**) was written by J. Safranek (then @ BNL).

- **LOCO** uses as input the orbit response matrix ***M*** giving the change in beam position with changes in steering magnet kicks,

$$\mathbf{u} = \boldsymbol{M}\,\theta$$

where
  - **u** is the beam position vector, $\mathbf{u}^T = (x_1, x_2, x_3, \ldots, y_1, y_2, \ldots y_N)$
  - θ is the kick vector, $\theta^T = (\theta_{x1}, \theta_{x2}, \ldots, \theta_{y1}, \theta_{x2}, \ldots \theta_{yM})$

- **LOCO** analyzes a measured response $\boldsymbol{M}^{meas}$ and tries to calibrate the linear optics of the machine by adjusting the model response $\boldsymbol{M}^{mod}$ via BPM gains, corrector calibration factors,…

LOCO principle (ii)

- For linear optics, ***M*** can be written
  - Closed orbit :

$$M_{ij} = (\beta_i \, \beta_j)^{1/2} \cos(|\mu_i - \mu_j| - \pi Q) / (2 \sin(\pi Q))$$

  - Trajectory :

$$M_{ij} = (\beta_i \, \beta_j)^{1/2} \sin(\mu_i - \mu_j) \qquad \mu_i > \mu_j$$

$$M_{ij} = 0 \qquad \mu_i < \mu_j$$

- ***M*** holds a lot of optics information, but in a complicated "form".
- A measurement of ***M*** is simple and non-destructive.

LOCO was used successfully in many places (NSLS, ALS, PEP-II,…)

LOCO method (i)

- **LOCO step 1** : build a vector **V** with elements

$$\mathbf{V_k} = (\boldsymbol{M}_{\mathbf{ij}}^{\mathbf{meas}} - \boldsymbol{M}_{\mathbf{ij}}^{\mathbf{mod}})/\sigma_i \qquad \text{for all i,j}$$

where :

- **meas** and **mod** refer to the measured and the machine model response matrices.
- σ_i is the i^th BPM error/noise RMS.

LOCO goal : vary $\boldsymbol{M}_{\mathbf{ij}}^{\mathbf{mod}}$ in order to minimize the norm of vector **V**,

$$|| \mathbf{V} || = \Sigma\, \mathbf{V_k}^2 = \text{minimum} \sim (\text{no. elements of } \mathbf{V} - \text{no. free parameters})$$

LOCO method (ii)

❑ **LOCO step 2** : evaluate the dependence of V on various parameters c_l and define the (linearized) sensitivity matrix $\boldsymbol{S}$

$$\boldsymbol{S}_{kl} = \partial \boldsymbol{V}_k / \partial c_l$$

Example :

- If c_l is a BPM gain : $\boldsymbol{S}_{kl} = -\boldsymbol{M}_{ij}^{mod}/\sigma_i$
- If c_l is a corrector calibration : $\boldsymbol{S}_{kl} = \boldsymbol{M}_{ij}^{mod}/\sigma_i$
- If c_l is a MAD parameter : $\boldsymbol{S}_{kl} = (\boldsymbol{M}_{ij}^{mod}(c_l + \Delta c_l) - \boldsymbol{M}_{ij}^{mod}(c_l))/(\Delta c_l\ \sigma_i)$
- …

LOCO method (iii)

❑ **LOCO step 3** : solve for the parameter increment vector $\Delta\mathbf{c}$:

$$\mathbf{V} + \Delta\mathbf{V} = \mathbf{V} + \mathbf{S}\,\Delta\mathbf{c} = 0$$

- This equation is identical to an orbit correction equation !
- Can be solved with the same least-square algorithms : SVD, MICADO..
- Matrix ***S*** is actually **rank deficient (singular)** :

  There are an ∞ no. of solutions because it is always possible to multiply all BPM and corrector gains by an arbitrary scale factor !

⬇

SVD is used to solve the system and the smallest eigenvalues, associated to the singularities, are removed.

LOCO method (iv)

- **LOCO step 4** : update the model (→ ***V***) and parameter vector **c**.

Since ***M*** is not a linear function of quadrupole gradients,… the procedure must be iterated until the results converge.

- The horizontal scales (BPMs + correctors) can finally be fixed using the horizontal dispersion, provided one knows the energy or RF frequency changes.

LOCO program (I)

- LOCO consists of a set of FORTRAN programs that interact with MAD
- Outline of the procedure :

1) The USER has to generate some input files :
   - Measured response matrix
   - Parameter definition file :
     - BPM gains, quality flags, roll angles.
     - Corrector names, kicks, flags, roll angles.
     - MAD parameters : names, input values, increments.
2) LOCO automatically generates MAD scripts to
   - loop through the correctors and “bump” each one in turn.
   - loop through MAD parameters.
3) LOCO uses the MAD output to generate $\boldsymbol{M}^{\mathbf{mod}}$ for all parameter settings.
4) The main LOCO program performs the fit.
5) LOCO automatically generates a new input command file from its output to prepare the next iteration.

LOCO program (ii)

- I took over the software and made some modifications, respectively adapted it to fit with the SPS MAD sequence :
  - single plane BPMs (I later learned that there was already a modified version to handle this…).
  - added a flag (+ MAD command generation) to switch from a ring to a line.
  - added a filter on the BPM gains to automatically reject very bad guys for the next iteration.
  - added the possibility to renormalize the BPM and corrector gain scales between 2 iterations.
  - wrote a program to generate simulated input from MAD data with BPM calibration errors and noise, corrector calibration errors.
  - …

Matrix sizes…

- Consider a ring with **N** BPMs and **M** correctors per plane. The minimum size of matrix ***S*** is (only BPM + correctors to calibrate) :

  - no coupling between H & V : $(2 \times N \times M) \times (2 \times (N + M))$
  - with coupling : $(4 \times N \times M)$ x $(2 \times (N + M))$

    - SPS : N = 113 , M = 108 ~ 49000 x 442
      → 22 10^6 elements
    - LHC : N = 500 , M = ~ 250 ~ 500000 x 1500
      → 750 10^6 elements !!!

Memory + numerical accuracy (?) :
For very large machines, one has to restrict to a fraction of the correctors / split the data.
There is anyhow some redundancy in the correctors..

SPS measurements

- **Measurement conditions in the SPS :**
  - LHC type beam on SC 538, injection at 26 GeV.
  - Time in the cycle : 2010 ms / 66 GeV.
  - Corrector kicks : +20 & –20 μrad (± 2 mm peak orbit changes).
  - Transparent measurement during physics.
  - Tunes = (26.76, 26.83) – i.e. not our nominal tunes of (26.62,26.58)
  - Bumped 18 correctors in H, 21 correctors in V.
  - All bumped correctors in sextants 1 & 2.

Fit sequence

- **Fit model input :**
  - The nominal SPS model with tunes of (26.62,26.58) to “simulate” an optics error !
  - Free strength parameters :
    - QD,QF1 & QF2 main quad strengths.
    - 6 LQS (skew) quad strengths → **attempt to model the coupling**

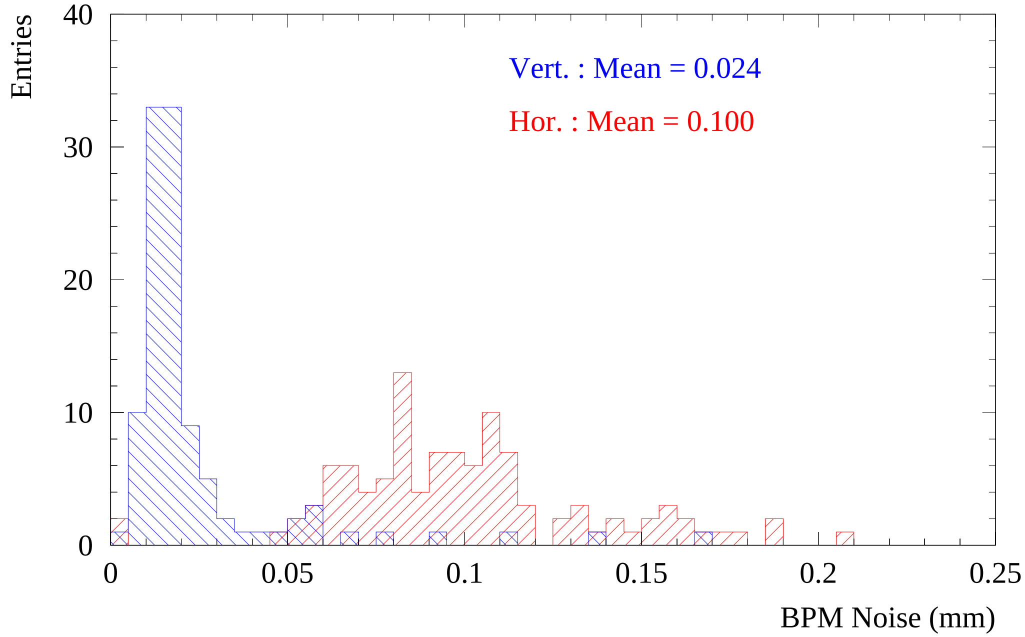
- **Fit sequence :**
  - 1 - Uncoupled fit to adjust each plane separately.
  - 2 - Coupled fit with :
    - Energy changes between + and – kick.
    - Coupling (LQS).
    - Corrector roll angles.

- **Results presented here** : **coupled fit without corrector roll**.

BPM noise

The noise is estimated from the RMS position change over 14 reference orbits acquired during the 90 minutes of measurements :

- vertical → excellent - 24 μm on average
- horizontal → dominated by momentum fluctuations !
- does not include errors from non-linearities…(obviously !)



Orbit resolution is 10 μm !

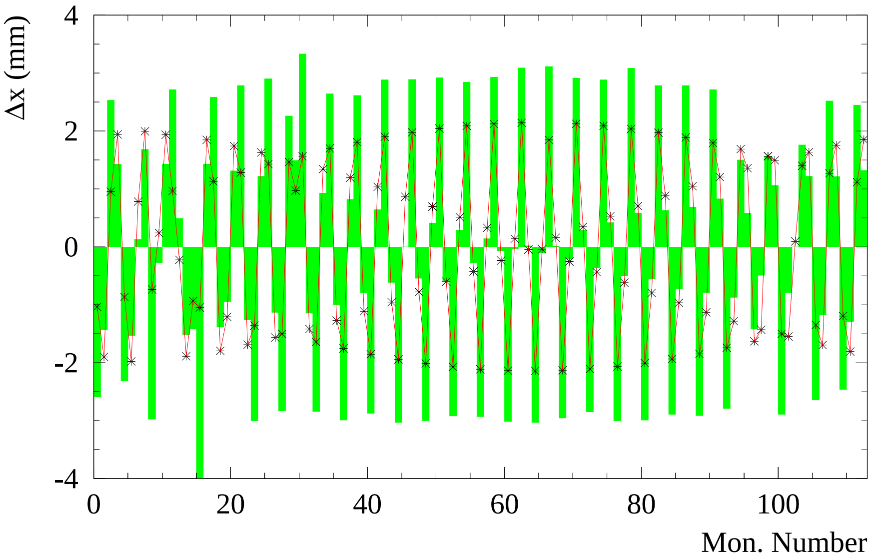
Before fit…

Examples of "in plane" data : response θ^+ - response θ^-

The large amplitude error is due to the orbit factor sin(πQ) :

$$\sin(26.6\,\pi)/\sin(26.8\,\pi) \sim 1.6$$

Histogram : raw data **(*) + line : model , tunes = (26.62,26.58)**



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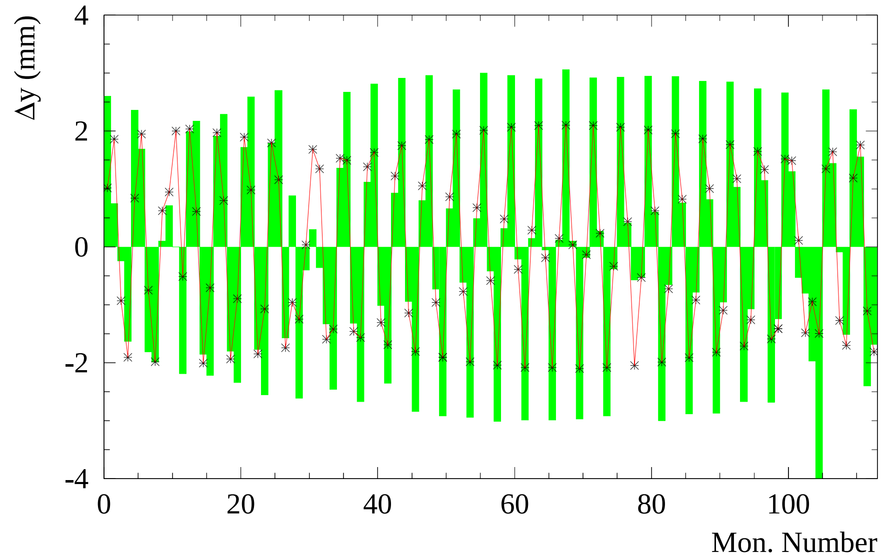

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7 LOCO iterations later…

Agreement data-fit model :

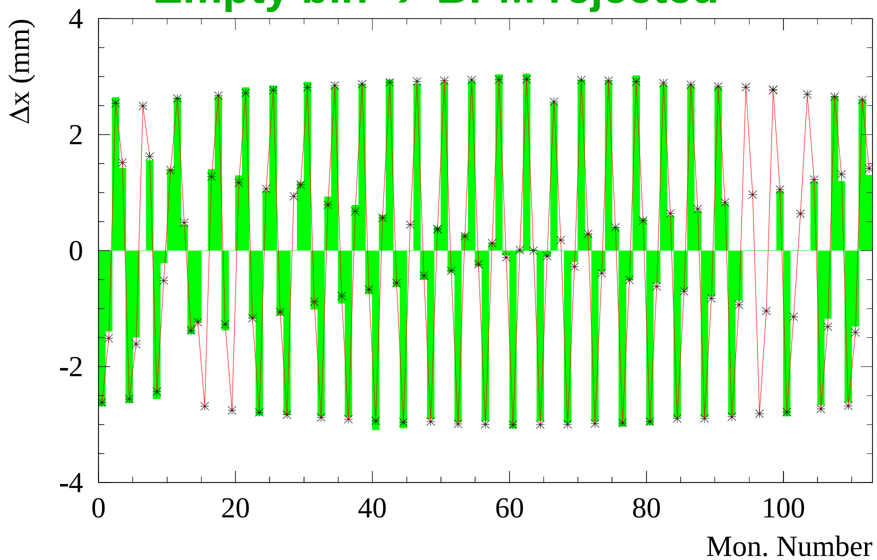
- H plane : RMS difference ~ 80 μm → expect 140 μm
- V plane : RMS difference ~ 55 μm → expect 34 μm

} $\sqrt{2} \times$ noise

Fit tunes = (26.762, 26.826) , ~ no visible β-beating !

Histogram : gain corrected data
Empty bin → BPM rejected

(*) + line : fit model with calibrated kick



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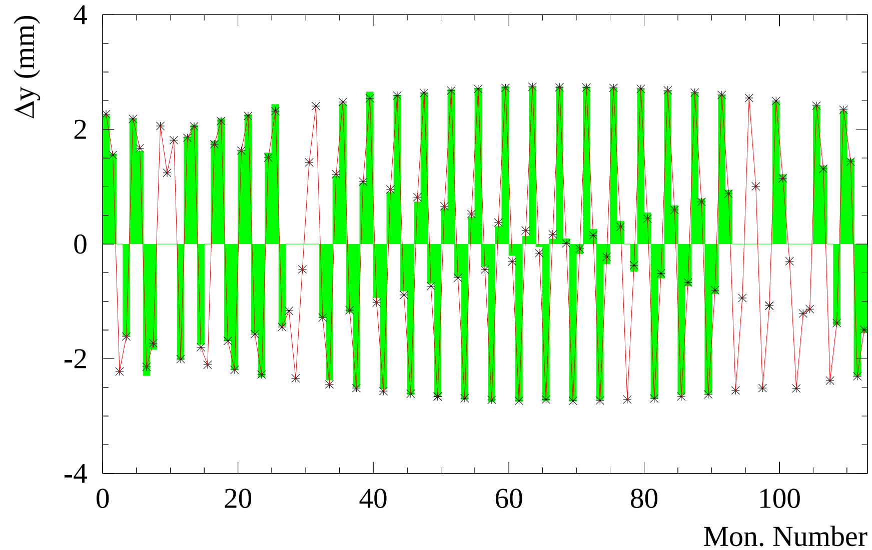

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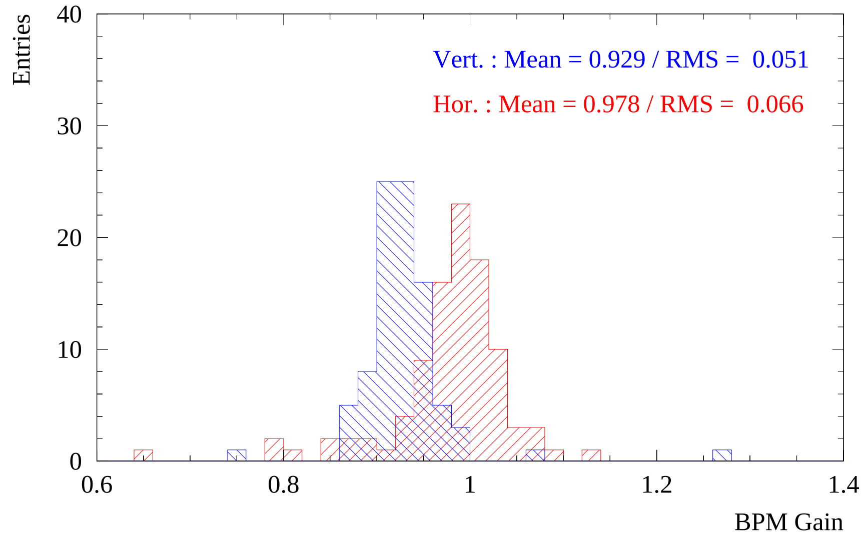
Calibration factors

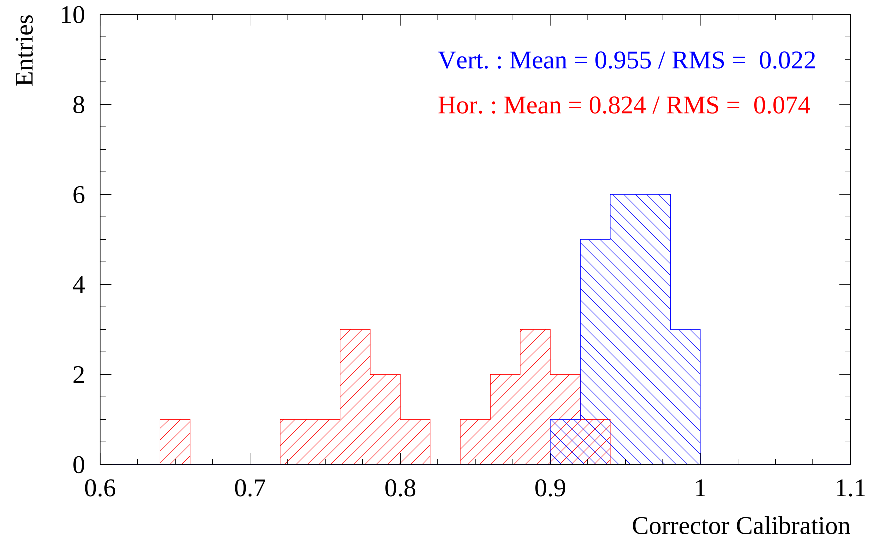
Calibration factors : **36 out of 226 monitors rejected !!**

- H plane : **BPM gains (re)normalized to dispersion !**
  Corrector gains very low + large RMS.
- V plane : calibrations ~ OK.

Estimated BPM gain accuracy is < 1% in both planes !

(<-- > splitting sample in 2 sub-samples – agrees with simulation)





Horizontal dispersion

The dispersion :

• not included in the fit, since it also depends on the bending (errors).

• can be used to check the model and set the BPM scale.

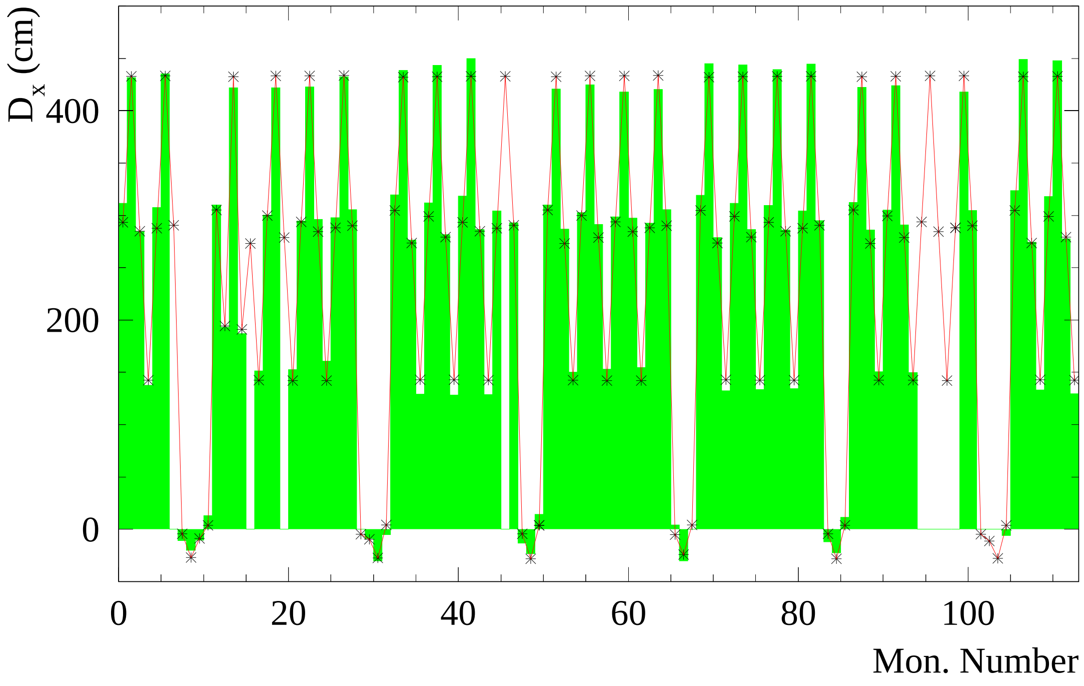
Data-fit :

→ Not a perfect match

→ Sextant dependent amplitude "beating"

Bend errors ?

(*) + line : fit model **Histo : gain corrected data**

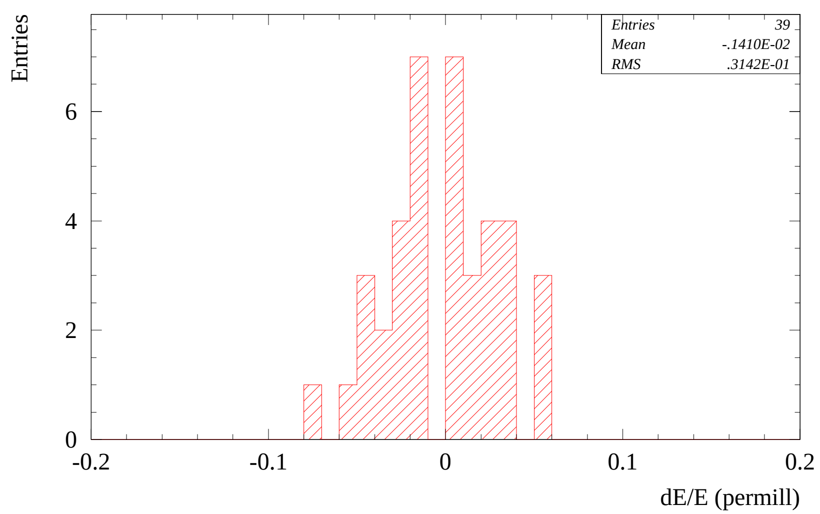


Momentum fluctuations

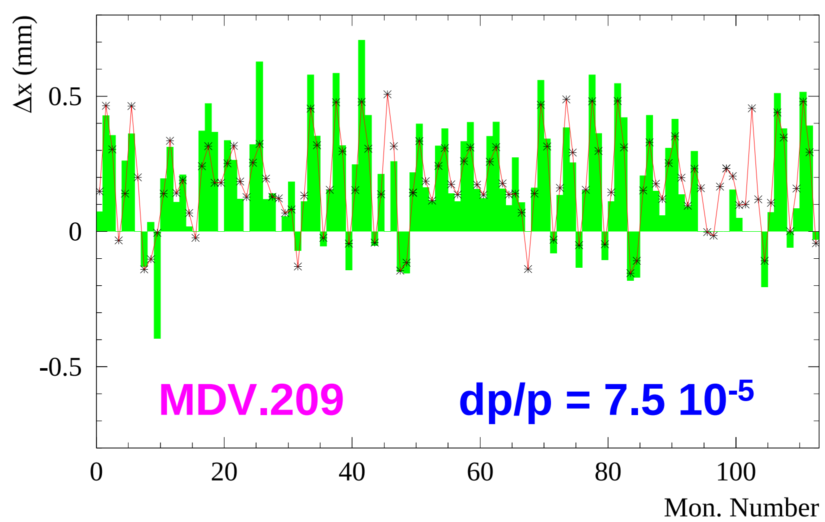
Coupling V kicks → H plane :

- reveals cycle to cycle momentum fluctuations
  - → dominant effect, also for the “noise” estimate !
- RMS fluctuation is ~ **$3\ 10^{-5}$**.

dp/p distribution



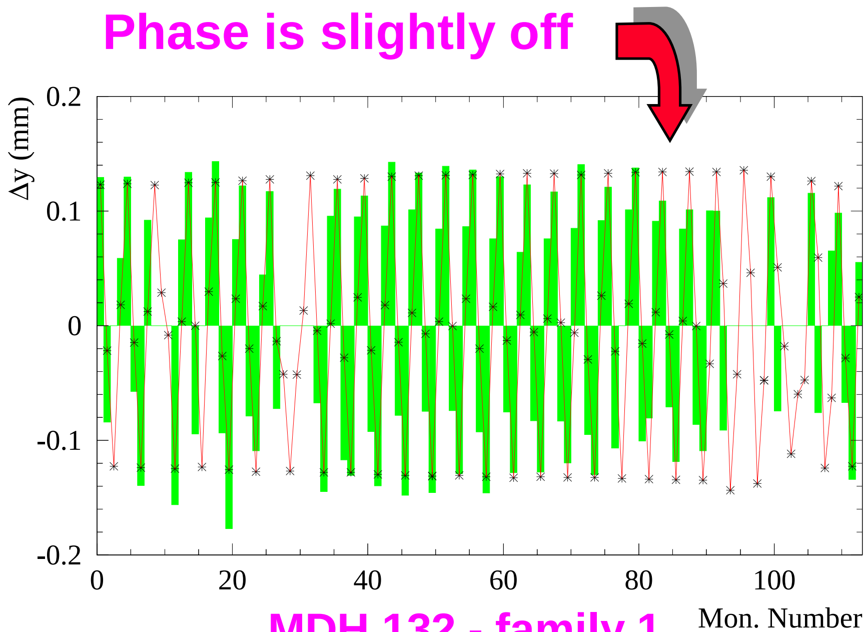
(fit also includes coupling)



Coupling

Coupling H kicks → V plane :

- systematic coupling visible.
- not perfectly “described” with the LQS quads – **phase shift.**
- correctors come in 2 “families”, **90 degrees out of phase** :
    - family 1 : “larger” coupling
    - family 2 : “smaller” coupling



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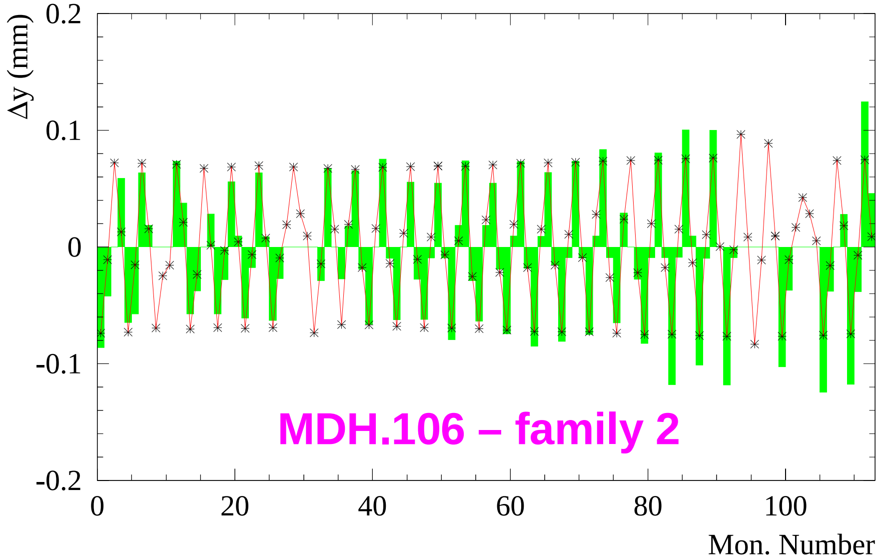


Some things don't fit (yet)…

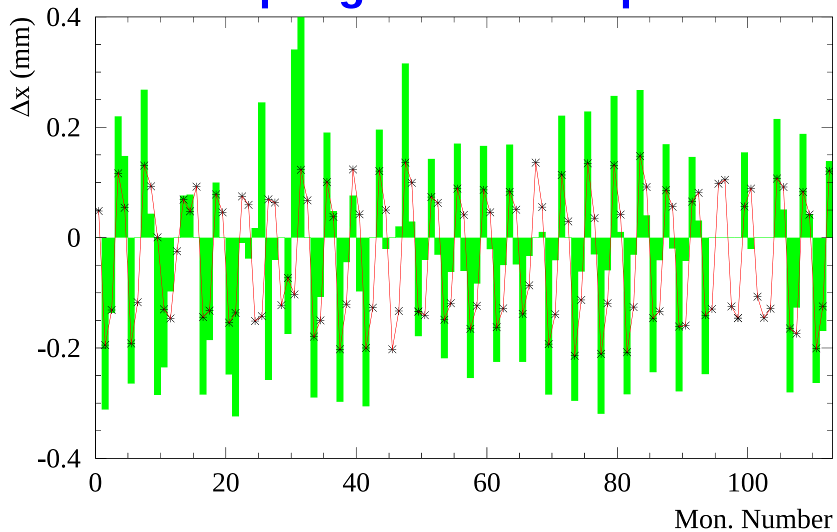
Coupling V kicks → H plane :

- some correctors have poor fits (see below).
- large(r) BPM errors in H – does not help…

Vertical dispersion :

- predicted dispersion due to coupling is somewhat large.

Coupling V kick → H plane



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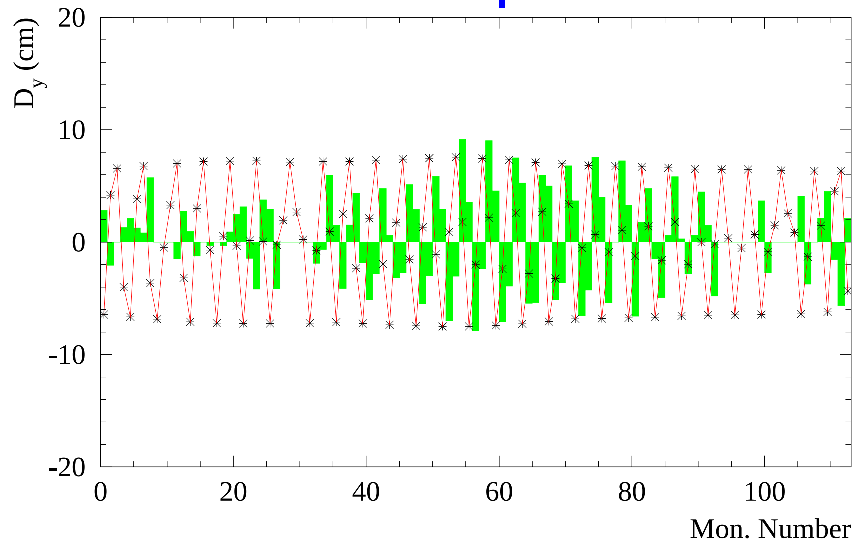
Vertical dispersion



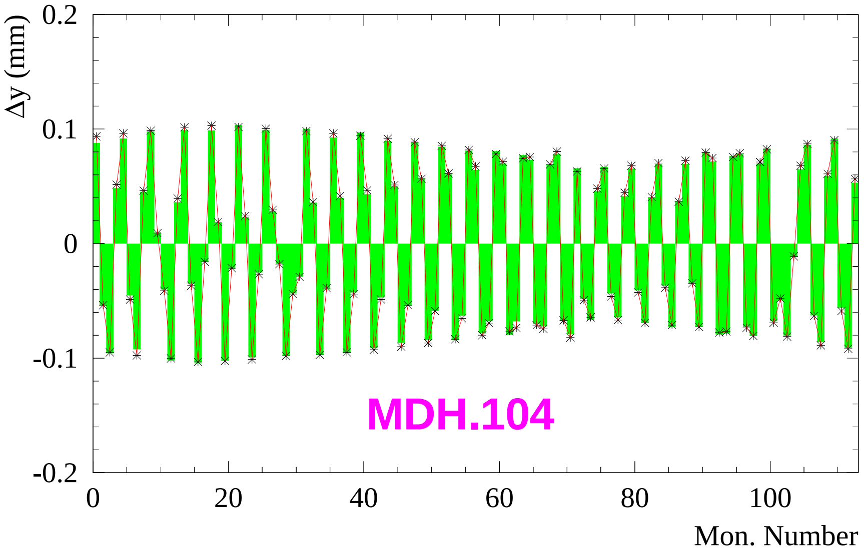
Coupling test (i)

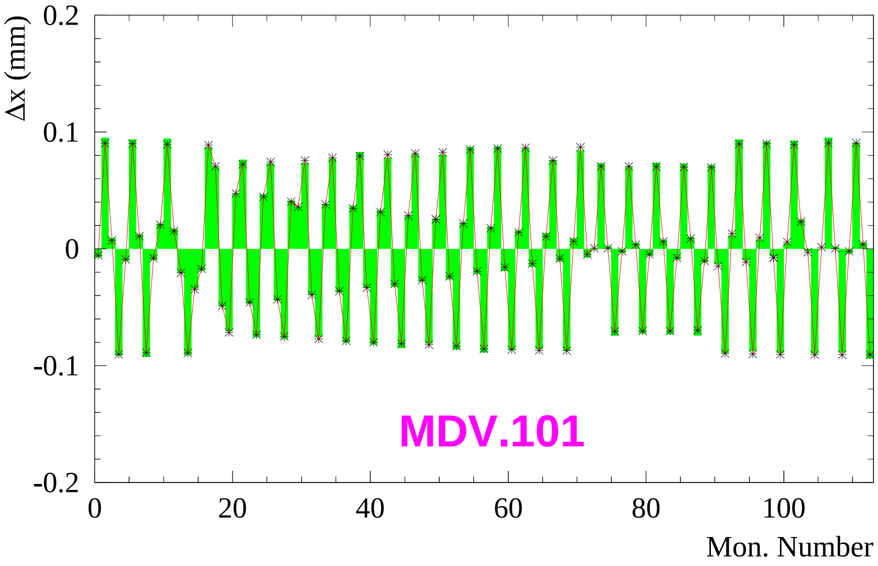
LQS simulation test :

- 3 LQS set to non zero strengths of 0.5 to 1.5 10^{-3} m^{-2}.
- Simulated orbit response with MAD for ~ 20 correctors/plane.
- BPM calibration errors of 5%, cor. calibration errors of 2%, BPM noise of 30 µm.

LOCO fit works perfectly – strengths accurate to < 4 10^{-5}

Histo : input model **(*) + line : fit model**





Coupling test (ii)

Rolled quadrupole simulation :

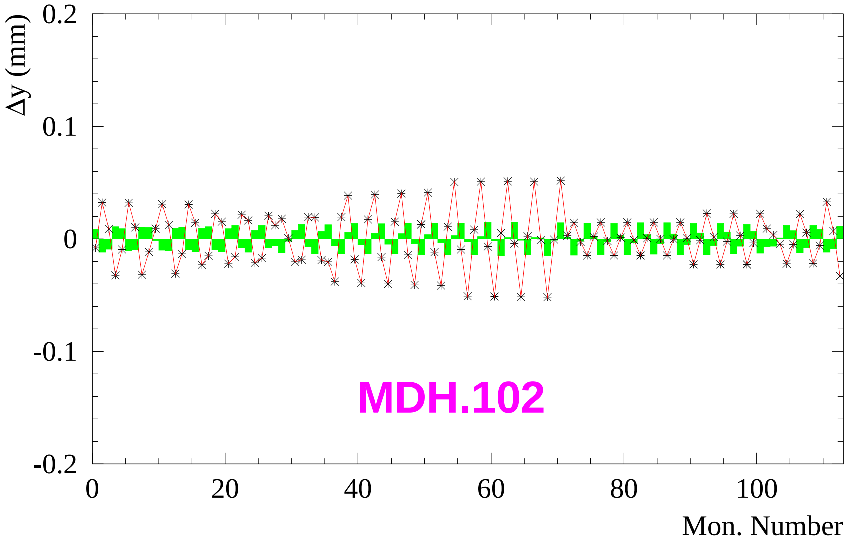
- Rolled quad **QF1.420** in LSS4 by **15 mrad** (small D_x).

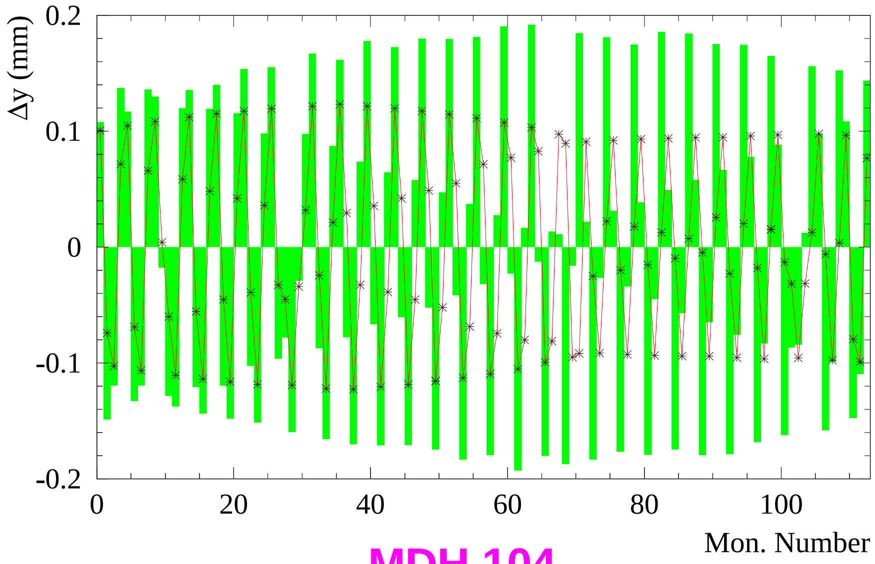
LOCO fit result :

- **It seems not possible to correct orbit coupling with LQS quads**.
- Closest tune approach : CTA = 0.01 → CTA = 0.004
- CTA can be reduced to ~ 0.001 using coupling knob (real comp.)

  → **worse for the orbit coupling !**

Histo : input model **(*) + line : fit model**





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Strength stability

- Estimate of the strength stability in the fits :
  - split the corrector sample in 2 and refit separately

	K (10^{-2} m^{-2})	Max |dK| (10^{-2} m^{-2})
QD	-1.47026	**$7\ 10^{-4}$**
QF1	1.47326	$2\ 10^{-4}$
QF2	1.47329	$13\ 10^{-4}$
LQS	0.04 **to** 0.11	**0.04**

  - QD & QF : excellent
  - LQS : larger changes – reflect the fit imperfections for coupling

Summary & Outlook

- LOCO works very well and already reveals interesting effects …
- Further analysis of this data :
  - improve noise estimate (mainly H plane).
  - check orbit coupling effects …
  - refit with corrector roll angles : for the moment LOCO clearly tries to compensate the imperfect coupling fit by rolling the H correctors in a strange fashion.
  - …
- Test LOCO in the LHC transfer lines – the machinery is set up for TI8.
- Test LOCO on TT10 ?
- LHC…..

MD proposals

- I will adapt the SPS orbit program to perform the measurements automatically (following a given corrector list).

- (At least) 2 measurements under the same conditions :
  - can probably be done parasitically to the physics program !
  - gives an idea of the reproducibility.

- Studies on the p2 cycle with LHC test beam :
  - perform one reference measurement.
  - create controlled beta-beat and re-measure.
  - create controlled coupling and re-measure.
  - other ideas ?
  - 1 day should be sufficient for this program..

- Tests on TT10 ?

Phase advance fits (i)

For measurements of the phase advance between BPMs using single kick/AC dipole techniques one could adapt the LOCO philosophy !

- Consider :
  - the vector **m** holding the phase advance from one BPM to the next,

    $\mathbf{m}^T = (\Delta\mu_1, \Delta\mu_2, \ldots, \Delta\mu_N)$
  - the difference vector $\Delta\mathbf{m} = \mathbf{m}^{meas} - \mathbf{m}^{mod}$ between the measured and the modeled phase advance.

- We can define the sensitivity matrix ***S*** of the phase advance on some parameters g_j (gradients…) :

$$S_{ij} = \partial \mathbf{m}_i / \partial g_j$$

which can be evaluted from MAD in a similar fashion than for the orbit response…

Phase advance fit (ii)

- To minimize the vector $\Delta\mathbf{m}$ we must solve the linearized equation :

$$\Delta\mathbf{m} + \mathbf{S}\,\Delta\mathbf{g} = 0$$

which is again identical to the LOCO, orbit correction… equations.

- Since the problem is again non-linear, we must update the model, $\mathbf{g} \rightarrow \mathbf{g} + \mathbf{Dg}$, and iterate until it (hopefully) converges.

- The advantage for LARGE machines is that the size of ***S*** is :

(Number of BPMs x Number of gradients)

since the method does not depend on BPM and corrector magnet calibrations.